

Quantitative Finance Formulas

Interest accumulation: $FV = PV + I$

Simple interest: $FV = PV (1 + i \cdot t)$

Compound interest: $FV = PV (1 + i)^t$

Simple discount: $D = Fv \cdot d \cdot t$

I=Interest; P=Principal; i=interest rate

t=number of periods

Effective rates conversion:

$$i_L = (1 + i_S)^{L/S} - 1; i_S = (1 + i_L)^{S/L} - 1$$

Relation between nominal and effective rates:

$$i_A(m) = m[(1 + i_A)^{1/m} - 1]$$

Continuous compounding:

Nominal rate: $\delta = \ln(1 + i_A)$

Future Value: $S = Pe^{\delta t}$

Present Value: $P = Se^{-\delta t}$

Present value of a n payment annuity immediate of

1 per period: $a_{\bar{n}|i} = \frac{1 - (1+i)^{-n}}{i}$

Accumulated value of a n payment annuity immediate of 1 per period:

$$s_{\bar{n}|i} = \frac{(1+i)^n - 1}{i} = a_{\bar{n}|i}(1+i)^n$$

Present value of annuity due:

$$\ddot{a}_{\bar{n}|i} = 1 + a_{\overline{n-1}|i} = a_{\bar{n}|i}(1+i)$$

Accumulated value of annuity due:

$$\ddot{s}_{\bar{n}|i} = s_{\bar{n}|i}(1+i)$$

Present value of deferred annuity:

$${}_k|a_{\bar{n}|i} = a_{\bar{n}|i}(1+i)^{-k}$$

Accumulated value of deferred annuity:

$${}_k|s_{\bar{n}|i} = s_{\bar{n}|i}$$

Forborne annuities

$$FV = R \cdot S_{n|p}(1+i)^p$$

p- number of intervals between the last payment and FV.

Present value of perpetuity immediate: $a_{\infty|i} = \frac{1}{i}$

Increasing arithmetic progression:

$$(C - h)a_{\bar{n}|i} + h(IA)_{\bar{n}|i}; \quad (IA)_{\bar{n}|i} = \frac{\ddot{a}_{\bar{n}|i} - n(1+i)^{-n}}{i}$$

Decreasing arithmetic progression:

$$(D - h)a_{\bar{n}|i} + h(DA)_{\bar{n}|i}; \quad (DA)_{\bar{n}|i} = \frac{n - a_{\bar{n}|i}}{i}$$

Geometric progression: $C \frac{1 - r^n(1+i)^{-n}}{1+i-r}$

M^{thly} payable annuity:

$$a_{\bar{n}|i}^{(m)} = a_{\bar{n}|i} \frac{i}{i^{(m)}}; \quad s_{\bar{n}|i}^{(m)} = s_{\bar{n}|i} \frac{i}{i^{(m)}}$$

Leasing:

Lease payment = PMT + I

PV = PMT $a_{\bar{n}|i}$, I = RV · i

Leasing (for an annuity immediate):

$$Vc = E + Ra_{\bar{n}|i} + RV(1+i)^{-n}, \text{ where}$$

Vc: value of the contract; E: entry value

RV = residual value; PMT = periodic payment

Linear Interpolation:

$$R_n = R_1 + [(R_2 - R_1) / (t_2 - t_1)] \cdot (t_n - t_1)$$

R_n - unknown rate

R₁ and R₂ - two known